California Math Multilingual Learner Report: Illustrative Mathematics 6-8

CALIFORNIA CURRICULUM COLLABORATIVE

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Illustrative Mathematics 6-8

1a Report

Criterion 1	Materials consistently provide opportunities for simultaneous content, math practices, and language development.
Indicator 1a	Materials describe major math language goals (informed by language demands, language forms and functions, language objectives) at the lesson and/or unit level.

Each lesson includes Learning Goals found within the Lesson Preparation. The language goals/objectives are explicitly tied directly to the content objectives and connected and embedded within the Learning Goals to be transparent to teachers and students. Examples include:

Grade 6:

- Unit 2, Introducing Ratios, Lesson 3: Recipes, Learning Goals, provide the math language goal, “Explain equivalent ratios (orally and in writing) in terms of different sized batches of the same recipe having the same taste.” This particular learning goal is an explicit language goal that requires students to use language to describe ratios orally and in writing, using a specific language form.
- Unit 5, Arithmetic in Base Ten, Lesson 15: Making and Measuring Boxes, Learning Goals provides the math language goal, “Apply operations with decimals to calculate the surface area of paper boxes. Describe (orally) sources of measurement error, and justify an appropriate level of precision for reporting the answer. Measure and compare (orally and in writing) the dimensions of paper boxes.” This learning goal is explicit in the function (measure and compare, describe, calculate) and form (orally and in writing).
- Unit 6, Expressions and Equations, Lesson 8: Equal and Equivalent, Learning Goals, provide the math language goal “Explain (in writing) that some pairs of expressions are equal for one value of their variable but not for other values. Justify (orally, in writing, and through other representations) whether two expressions are “equivalent”, i.e., equal to each other for every value of their variable.” The learning goals provide explicit language targets, with the demands of explaining and justifying both orally and in writing, with a targeted language of “equivalent.”

Grade 7:

- Unit 5, Rational Number Arithmetic, Lesson 10: Multiply, Learning Goals, provide the math language goal, “Identify multiplication expressions that are equal, and justify (orally) that they are equal. Multiply rational numbers, including multiplication expressions with three factors, and explain (orally and in writing) the reasoning.” The language goal is explicit with the domains, orally and in writing, the language function (identify, justify and explain), and the form and desired language.
- Unit 7, Angles, Triangle, and Prisms, Lesson 17: Building Prisms, Learning Goals, provide the math language goal “Compare and contrast (orally) triangular prisms, including comparisons of their height, cross sections, surface area, and volume. Compose two triangular prisms into a new prism, and describe (orally and in writing) the composite shape.” The language goal is explicit with the domains, orally and in writing, the language function (compare and describe), and the form and desired language of what aspects of the prisms compared and the desired vocabulary.
- Unit 8, Probability and Sampling, Lesson 11: Comparing Groups, Learning Goals, provide the math language goal, “Calculate the mean and mean absolute deviation for a data set, and interpret (orally) these measures. Compare and contrast (orally and in writing) populations represented on dot plots in terms of their shape, center, spread, and visual overlap. Justify (in writing) whether two populations are “very different” based on the difference in their means expressed as a multiple of the mean absolute deviation.” The learning goals provide explicit language targets, with the demands of writing, interpreting, comparing, contrasting, and justifying both orally and in writing.

Grade 8:

- Unit 3, Linear Relationships, Lesson 6: More Linear Relationships, Learning Goals, provide the math language goal, “Describe (orally and in writing) how the slope and vertical intercept influence the graph of a line. Identify and interpret the positive vertical intercept of the graph of a linear relationship.” The language goal is explicit in the domains of oral and written language as students describe the function and interpret the vocabulary terms slope, intercept, and line.
- Unit 4, Linear Equations and Linear Systems, Lesson 3: Balanced Moves, Learning Goals, provide the math language goal, “Compare and contrast (orally and in writing) solution paths to solve an equation in one variable by performing the same operation on each side. Correlate (orally and in writing) changes on hanger diagrams with moves that create equivalent equations.” The language goal is explicit with the domains, orally and in writing, the language function (compare and describe), and the form and desired language of what aspects of the prisms compared and the desired vocabulary.
- Unit 7, Exponents and Scientific Notation, Lesson 15: Adding and Subtracting with Scientific Notation, Learning Goals, provide the math language goal,“Generalize (orally and in writing) a process of adding and subtracting numbers in scientific notation and interpret results in

context." The language goal is explicit in the domains (orally and in writing) and the function of generalizing a process by using the form of interpreting the results in context.

1b Report

Criterion 1	Materials consistently provide opportunities for simultaneous content, math practices, and language development.
Indicator 1b	Materials describe the math language progression for how students will bridge between everyday and mathematical ways of communicating.

The materials provide examples that show how the materials intentionally develop language over time, with plans to bridge between everyday and formal ways of communication, and/or how to support mathematical ways of thinking, however the materials do not explicitly call out the language progression. The Progression of Disciplinary Language and New Terminology tables are included for most units of each grade within the unit narrative, highlighting lessons across the unit for the 3 most common uses of language for mathematical purposes. Other examples include:

Grade 6:

- Unit 1, Area and Surface, Lesson 1: Tiling the Plane, Activity 1.2, More, Red, Green, or Blue?, Access for English Learners illustrates how students bridge between everyday and mathematical ways of communicating. "Conversing, Speaking, Listening: Math Language Routine 2 Collect and Display. This is the first time Math Language Routine 2 is suggested as a support in this course. In this routine, the teacher circulates and listens to students talk while writing down the words, phrases, or drawings students produce. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students' words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations. Design Principle(s): Support sense-making How It Happens: 1. After assigning students to work on Pattern A or B, circulate around the room and collect examples of language students are using to compare areas of polygons. Focus on capturing a variety of language describing the relationship between the size of two shapes, comparing overall quantities of shapes to equivalent areas of other shapes, and comparing relevant quantities in a hexagon. Aim to capture a range of student language that includes formal, precise, complete ideas and informal, ambiguous, and partial ideas. Plan to publicly update and revise this display

throughout the lesson and unit. If pairs are stuck, consider using these questions to elicit conversation: “How many green triangles, blue rhombuses, and red trapezoids are in each pattern?”, “Three triangles is equivalent to how many trapezoids?”, and “Which shapes make up a hexagon?” If using the applet, have pairs use the applet together. Check that students focus on how many of each shape comprise the pattern by hiding, moving, and turning the shapes. 2. Create a display that includes visual representations of the words and phrases collected. Group language about Pattern A on one side of the display and language about Pattern B on the other side. 3. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and then have students move on to discussing other aspects of the activity. Continue to publicly update and revise the display throughout the lesson and unit.” This example shows how the materials sometimes include a plan to bridge between everyday and formal ways of communication.

- Unit 2, Introducing Ratios, Course Guide, Narrative, prepares teachers for the language progression within each unit for students to bridge between everyday and mathematical thinking. “One of the principles that guided the development of these materials is that students should encounter examples of a mathematical concept in various contexts before the concept is named and studied as an object in its own right. The development of ratios, equivalent ratios, and unit rates in this unit and the next unit is in accordance with that principle. In this unit, equivalent ratios are first encountered in terms of multiple batches of a recipe and “equivalent” is first used to describe a perceivable sameness of two ratios, for example, two mixtures of drink mix and water taste the same or two mixtures of red and blue paint are the same shade of purple. Building on these experiences, students analyze situations involving both discrete and continuous quantities, and involving ratios of quantities with units that are the same and that are different. Several lessons later, equivalent acquires a more precise meaning (MP6): All ratios that are equivalent to a : b can be made by multiplying both a and b by the same non-zero number (note that students are not yet considering negative numbers).” This example shows how the materials intentionally develop language in ways valued by disciplinary practices.

Grade 7:

- Unit 1, Scale Drawings, Course Guide, Narrative, Progression of Disciplinary Language prepares teachers for the language progression within each unit for students to bridge between everyday and mathematical thinking. “The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow where it was first introduced.” This evidence shows when teachers transition into specific mathematical language.

- Unit 6, Expressions, Equations, and Inequalities, Lesson 16: Interpreting Inequalities, Activity 16.2, Club Activities, Matching, Access for English Learners illustrates how students bridge between everyday and mathematical ways of communicating. “Reading, Representing: MLR2 Collect and Display. As students work, circulate and collect examples of words and phrases students use in their written response to "Explain your reasoning" for each question. Look for different ways students describe what the variable represents, how they know which number is the constant term, how they know which number should be multiplied by the variable, and the direction of the inequality symbol that makes sense for each context. Organize the phrases for each of these considerations and display for all to see. This will help students to focus on all of the important elements of the inequality they are assigned in the next activity, with language they can use in small group discussions. Design Principle(s): Support sense-making Maximize meta-awareness.” This example shows how the materials sometimes intentionally develop language over time and how to support mathematical ways of thinking.

Grade 8:

- Unit 1, Rigid Transformations and Congruence, Lesson 4: Making the Moves, Activity 4.2, Make That Move, Access for English Learners, illustrates how students bridge between everyday and mathematical ways of communicating. “Listening, Speaking: Math Language Routine 2 Collect and Display. This is the first time Math Language Routine 2 is suggested as a support in this course. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing students use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students’ words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students’ awareness of language used in mathematics conversations. Design Principle(s): Optimize output (for explanation); Maximize meta-awareness. How It Happens: As students describe the transformation of triangle ABC to their partner, listen for and collect vocabulary and phrases students use to describe the moves. Focus on capturing students using geometric language for describing reflections, rotations, and translations. If the speaker is stuck, consider asking these questions: “How did point A transform to A’?”, “Choose one of the points, lines, or angles and describe how it changed.”, and “Overall, does it look like the new triangle is a translation, rotation, or reflection of the original?” If using the applet, check for precision and labels as students place the new image under the transformation. Write students’ words on a visual display. Divide the display into 3 sections. Group language about Cards 1A and 2A on the left side of the display, language about Card 1B in the middle, and language about Card 2B on the right side. Record all language (whether precise, ambiguous, correct, or incorrect) in the appropriate column. as described by the students. Arrange students in groups of 2, and invite partners to discuss which words or phrases stand out to them. Prompt students by asking, “Are there any words or phrases

that stand out to you or don't belong in a specific column? Why?" Again, circulate around the room, collecting any additional words, phrases, and sketches onto the display. Students should notice that the left side consists of language describing translations, the middle consists of language describing reflections, and the right side consists of language describing rotations. Select 3–4 groups to share their ideas with the class. Invite students to demonstrate their reasoning with the applet or tracing paper and be sure to modify the display accordingly. Use this discussion to clarify, revise, and improve how ideas are communicated and represented. If students are still using vague words (e.g., move, flip, mirror image, etc.), reinforce the precise geometric terms (e.g., transformation, translation, rotation, reflection, etc.). Ask students, "Is there another way we can say this?" or "Can someone help clarify this language?" Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and be sure to update the display throughout the remainder of the lesson." This example shows how the materials sometimes intentionally develop language over time with a plan to bridge between everyday and formal ways of communication, and how to support mathematical ways of thinking.

- Unit 5, Functions and Volume, Lesson 1: Inputs and Outputs, Lesson Narrative illustrates how students bridge between everyday and mathematical ways of communicating. "This is the first of two lessons introducing students to functions, developing the idea of a function as a rule that assigns to each allowable input exactly one output. The word function is not introduced until the second lesson. In future lessons, students will expand on this definition as they work with different representations of functions. In the first classroom activity students take turns guessing each other's rules from input-output pairs. In the second activity students use rules represented by input-output diagrams to fill out a table with inputs and associated outputs. In each table, the first input-output pair is identical, illustrating that a single pair is insufficient for determining a rule. The last table returns to the topic of the warm-up and introduces the idea that not all inputs are possible for a rule." This example shows how the materials intentionally develop language in ways valued by disciplinary practices.

2a Report

Criterion 2	Materials provide tasks that require students to make meaning through collaboration by interpreting and producing language.
Indicator 2a	Tasks in materials require students to make meaning through collaboration.

The materials reviewed for Illustrative Mathematics include tasks that require students to make meaning through collaboration. Supports for teachers are found in the Launch and Activity Synthesis of each lesson. Collaboration is often partner-driven and includes whole class discussion. Examples include:

Grade 6:

- Unit 2, Introducing Ratios, Lesson 4: Color Mixtures, Activity 4.2, Turning Green Launch, Activity Synthesis, includes tasks that require students to make meaning through collaboration. “Arrange students in groups of 2–4. (Smaller groups are better, but group size might depend on available equipment.) Each group needs a beaker of blue water and one of yellow water, one graduated cylinder, a permanent marker, a craft stick, and 3 opaque white cups (either styrofoam, white paper, or with a white plastic interior).” Groups are asked to mix the colors and record their observations and attempt to make the solution “bluer.” “After each group has completed the task, have the students rotate through each group’s workspace to observe the mixtures and diagrams. As they circulate, pose some guiding questions.” Students make meaning of the results of certain mixtures and how the color is affected by various ratios by making collaborative observations and hypotheses.
- Unit 3, Unit Rates and Percentages, Lesson 13: Benchmark Percentages, Activity 13.2, Liters, Meters and Hours, Activity Synthesis, includes tasks that require students to make meaning through collaboration. “Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After providing some independent think time, use this routine with successive pair shares to give students a structured opportunity to revise and refine their explanations for how to find 75% of any number. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “Can you explain how…”, “You should expand on...”, etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other strategies to find 75% of any number. Design Principles(s): Optimize output (for explanation) Maximize meta-awareness.” Students are provided the opportunity to make meaning through collaboration by to think, pair share, revise, and refine. This occurs by providing prompts to help clarify their thoughts.
- Unit 5, Arithmetic in Base Ten, Lesson 5: Decimal Points in Products, Activity 5.2, Fractionally Speaking, Launch, includes tasks that require students to make meaning through collaboration. Students collaborate to build the mathematical understanding that a number multiplied by (1/10) is the same as divided by 10. “Arrange students in groups of 2. Ask one student in each group to complete the questions for Partner A, and have the other take the questions for Partner B. Then ask them to discuss their responses, answer the second question together, and pause for a brief class discussion.” Through this collaborative activity, students make meaning of the results of different mathematical operations using whole numbers and fractions to strengthen the ideas of multiplication and division by solving problems independently and then discussing their responses.

Grade 7:

- Unit 3, Measuring Circles, Lesson 6: Estimating Areas, Activity 6.2, House Floor Plan, Access for English Learners, includes tasks that require students to make meaning through collaboration. “Conversing: MLR5 Co-Craft Questions. Display the diagram of the floor plan without revealing the task statement to students. Ask pairs of students to write a list of possible mathematical questions about the situation. Then, invite pairs to share their questions with the class. This will provide students with an opportunity to orient themselves to the context, ensuring that students understand the components of a floor plan, and also to produce the language of mathematical questions related to finding the area of irregular figures. Design Principle(s): Cultivate conversation; Support sense-making.” Students make meaning through collaboration as they work with peers to craft a list of possible questions and use problem solving skills when they use the floor plan diagram.
- Unit 8, Probability and Sampling, Lesson 11: Comparing 1 Groups, Activity Synthesis 11.1, Notice and Wonder: Comparing Heights, Activity Synthesis, includes tasks that require students to make meaning through collaboration. “Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the definitive difference in height does not come up during the conversation, ask students to discuss this idea.The next activity looks more closely at comparing these data sets. It is not necessary to have students calculate anything (mean, median, MAD, IQR) yet.” Students make meaning through collaboration as they look at dot plots.
- Unit 9, Putting It All Together, Lesson 4: Restaurant Floor Plan, Activity 4.2, Cold Storage, Launch, includes tasks that require students to make meaning through collaboration. Students explore the floor plans of a standard walk-in refrigerator and walk-in freezer along with the cost per month for each. Students answer how much would it cost to keep them both cold and to explain their reasoning. Access for English Learners, “Writing, Reading, Conversing: MLR5 Co-craft Questions. Begin by displaying only the initial text describing the context of the problem and the information about the monthly costs of standard refrigerators and freezers (i.e., withhold the scale drawing and question about the walk-in refrigerator and freezer). Ask students, “What mathematical questions can you ask about this situation?” Give groups 2–3 minutes to write down questions they have. As students share their questions, focus on questions that address how to evaluate costs in relationship to the volume of the refrigerator or freezer. This will help students understand the context and identify any assumptions they are making prior to solving the problem.” This example provides evidence on how the tasks require students to make meaning through collaboration.

Grade 8:

- Unit 3, Linear Relationship, Lesson 7: Representation of Linear Relationship, Activity 7.3, Calculate the Slope, Activity Synthesis, includes tasks that require students to make

meaning through collaboration. Students list the vertical change, horizontal change, slopes, describe procedures for finding slope, and write slope expressions from the graph using the letters u, v, s, and t. “Ask students to share solutions to the first three problems, and ensure that everyone understands why the correct answers are correct. There is no requirement that students simplify fractions; a student who comes up with 40/20 for graph C is correct. The convention of simplifying a fraction can be especially helpful if further calculations need to be made (or in this case, to give an immediate sense of the size of the number).Invite students to share the procedure they came up with. Ideally, they will share several versions of “Subtract the *y*-coordinates, subtract the *x*-coordinates, and then divide the difference in ’s by the difference in ’s.” Note that it is important to subtract the *x*-coordinates for the two points in the same order as the y -coordinates, that is (9 - 3)/(4 - 1) for graph A, not (9 - 3)/(1 - 4). For the last question, invite students to share the expressions they came up with. Acknowledge any response that is equivalent to the correct answer, but be on the lookout for expressions like v - t/u - s, or (v - t)/(s - u) which are incorrect for different reasons.” This example provides evidence on how the tasks require students to make meaning through collaboration.

- Unit 4, Linear Equations and Linear Systems, Lesson 5: Solving Any Linear Equation, Activity 5.1, Equation Talk, Activity Synthesis, includes tasks that require students to make meaning through collaboration. “Some students may reason about the value of *x* using logic. For example, in $-3x = 9$, the *x* must be -3 since $-3 = 9$. Other students may reason about the value of *x* by changing the value of each side of the equation equally by, for example, dividing each side of $-3x = 9$ by -3 to get the result $x = -3$. Both of these strategies should be highlighted during the discussion where possible. To involve more students in the conversation, consider asking as the students share their ideas: “Can you explain why you chose your strategy?” “Can anyone restate ___’s reasoning in a different way?” “Did anyone reason about the problem the same way but would explain it differently?” “Did anyone reason about the problem in a different way?” “Does anyone want to add on to ___’s strategy?” “Do you agree or disagree? Why?’” This example provides evidence on how the tasks require students to make meaning through collaboration.
- Unit 6, Association in Data, Lesson 5: Describing Trends in Scatter Plots, Activity 5.2, Fitting Lines, Access for English Learners, includes tasks that require students to make meaning through collaboration. “Speaking, Listening: MLR7 Compare and Connect. Use this routine when students present their strategies for drawing lines that fit the data well. Ask students to first identify “what is the same and what is different” about each approach. Draw students’ attention to the different ways the lines were constructed (e.g., connecting leftmost and rightmost points; drawing a line through as many points as possible; drawing a line where half the points fall above and below the line, etc.). In this discussion, emphasize the mathematical language used to make sense of the different ways to construct lines that fit the data well. These exchanges strengthen students’ mathematical language use and reasoning when constructing and analyzing lines that fit data points well. Design

Principle(s): Maximize meta-awareness." Students make meaning through collaboration as they discuss with peers different ways to construct lines that fit the data well.

2b Report

Criterion 2	Materials provide tasks that require students to make meaning through collaboration by interpreting and producing language.
Indicator 2b	Tasks in materials require students to make meaning by interpreting and producing mathematical language.

Supports for tasks that require students to make meaning by interpreting and producing mathematical language can be found in a variety of places, including the Access for English Language Learners. According to the IM Curriculum, What Is a Problem-Based Curriculum?, "Mathematical Practices- In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning. Principles for Mathematics Teaching and Learning - In order to learn mathematics, students should spend time in math class doing mathematics. Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience rather than the elements that must be taught before problem solving. Students should take an active role, both individually and in groups, to see what they can figure out before having things explained to them or being told what to do. Teachers play a critical role in mediating student learning, but that role looks different than simply showing, telling, and correcting." Examples include:

From the course materials for all grades:

- Course Guide, Narrative, Instructional Routines, Group Presentations, states "Some activities instruct students to work in small groups to solve a problem with mathematical modeling, invent a new problem, design something, or organize and display data, and then create a visual display of their work. Teachers need to help groups organize their work so that others can follow it, and then facilitate different groups' presentation of work to the class. Teachers can develop specific questioning skills to help more students make connections and walk away from these experiences with desired mathematical learning. For example, instead of asking if anyone has any questions for the group, it is often more productive to ask a member of the class to restate their understanding of the group's findings in their own words." This evidence describes an instructional routine where

students work collaboratively in groups to make meaning by interpreting and producing mathematical language.

Grade 6:

- Unit 2, Introducing Ratios, Lesson 2: Representing Ratios with Diagrams, Activity 2.3, Blue Paint and Art Paste, Access for English Learners, illustrates students making meaning interpreting and producing mathematical language. “Speaking, Writing: MLR2 Collect and Display. Circulate and listen to student talk during partner or group work, and display publicly common or important words and phrases (e.g., for every, the ratio of, for each) students are using. Refer back to this list, and ask students to clarify their meaning, explain how they are useful, and to reflect on which words and phrases help to communicate ideas more precisely. This will provide access to important language for students to use as they are needed. Design Principle(s): Support sense-making.” As the teacher collects and displays student language, the teacher asks students to interpret the language that is displayed and produce more precise mathematical language. This helps them develop the mathematical ideas and language around equations.
- Unit 6, Expressions and Equations, Lesson 6: Write Expressions Where Letters Stand for Numbers, Activity 6.2, Lemonade Sales and Heights, Access for English Learners, illustrates students making meaning interpreting and producing mathematical language. “Conversing, Representing, Writing: MLR2 Collect and Display. During small-group discussion, listen for and collect the vocabulary and phrases students use to describe how to find the values of the table and how the expression represents the situation (e.g., “the number of cups is twice the number of dollars”). Make connections between how similar ideas are communicated and represented in different ways (e.g., “How do you see ‘twice’ in the tape diagrams and expressions?”). Remind students to borrow language from the display as needed. This will help students to use academic mathematical language during paired and group discussions when writing expressions representing situations with an unknown quantity. Design Principle(s): Maximize meta-awareness.” Students interpret and produce language as they converse, represent, and write to describe how to find the values of the table and how the expression represents the situation.

Grade 7:

- Unit 1, Scale Drawings, Lesson 7: Scale Drawings, Activity 7.2, Sizing Up a Basketball Court, Access for English Learners, illustrates students making meaning, interpreting, and producing mathematical language. “Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have completed the first three questions, provide them an opportunity to refine their reasoning for the third question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language. For example, “How did you use the scale in your calculations?”, “Why did you multiply each measurement from the drawing by 2?”, and "A detail (or word) you could add is ___, because . . . ." Students can borrow ideas and language from each partner to refine and clarify their original explanation.

This will help students refine their own explanation and learn about other ways to find actual measurements by using the measurements from a scale drawing. Design Principles(s): Optimize output (for explanation) Maximize meta-awareness." Students interpret and produce language while they write, speak, and listen to refine their own explanation. Students learn other ways to find actual measurements by using the measurements from a scale drawing.

- Unit 7, Angles, Triangles, and Prisms, Lesson 11: Slicing Solids, Activity 11.3, Card Sort: Cross Sections, Access for English Learners, illustrates students making meaning interpreting and producing mathematical language. Students collaboratively sort cards using cross sections and solid figures. "Listening, Speaking, Conversing: MLR7 Compare and Connect. After students have sorted the images into groups that make sense to them, ask students to investigate each other's work by taking a tour of their visual displays. Facilitate discussion among students by asking questions such as, "What similarities or differences do you see in other groups' sorting as compared to your sorting?" or "What worked well while sorting the images?" Guide students to make connections between specific features of the images, such as the shape of a cross section and the cross section made by the cuts. This will help foster students' meta-awareness of the language as they compare or contrast the sorting of images." Students interpret the language and work of their peers while they listen and investigate the work while making connections.

Grade 8:

- Unit 6, Association in Data, Lesson 3: What a Point in a Scatter Plot Means, Activity 3.2, Weight and Fuel Efficiency, Access for English Learners, illustrates students making meaning interpreting and producing mathematical language. "Writing, Conversing: MLR5 Co-Craft Questions. Display only the table without revealing the graph or questions that follow. Ask students to work with a partner to write possible questions that could be answered by the data in the table. Invite 2–3 groups to share their questions with the class. Look to see whether/how the weights and fuel efficiencies of the vehicles shows up in students' questions. Next, reveal the questions of the activity. This routine helps students consider the context of this problem and to increase awareness about language used to talk about weights and fuel efficiencies. Design Principle(s): Maximize meta-awareness; Support sense-making." Students interpret and produce language while they write and converse possible questions using the data from the table.
- Unit 8, Pythagorean Theorem and Irrational Numbers, Lesson 13: Cube Roots, Activity 13.2, Cube Root Values, Access for English Learners, illustrates students making meaning interpreting and producing mathematical language. "Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Call on students to use mathematical language to restate and/or revoice the strategy (or strategies) presented. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original

statement. This will provide more students with an opportunity to produce language that describes strategies to figure out which two whole numbers each cube root lies between. Design Principle(s): Support sense-making Maximize meta-awareness." Students interpret and produce language while they speak and use benchmark whole numbers for locating cube roots on a number line.

3a Report

Criterion 3	Materials provide responsive language and collaborative supports that amplify mathematical language development.
Indicator 3a	Materials guide teachers to be responsive to students' current understandings in both content and language.

The materials provide support for teachers to be responsive to students' current understandings in content, but not language. These content supports are found at the Lesson level within activity synthesis, Examples include:

Grade 6:

- Unit 2, Introducing Ratios, Lesson 4: Color Mixing, Activity 4.1, Number Talk: Adjusting a Factor, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of ratios. "Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted the strategy choice. To involve more students in the conversation, consider asking: "Who can restate ___'s reasoning in a different way?" "Did anyone solve the problem the same way but would explain it differently?" "Did anyone solve the problem in a different way?" "Does anyone want to add on to ___'s strategy?" "Do you agree or disagree? Why?"' The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Differentiation is not evident based on the level of multi-language learners.
- Unit 4, Introducing Ratios, Lesson 1: Size of Divisor and Size of Quotient, Activity 1.1, Number Talk: Size of Dividend and Divisor, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of dividends and divisors. "Invite a couple of students to share their answers and strategies for each problem. Record and display their explanations for all to see. Refer to MLR 2 (Collect and Display). After evaluating all four expressions, ask students: "What do you notice about the value of each expression as the divisor (the number we use to divide) gets larger?" "Why is the value of the expression getting smaller each time?" Highlight explanations that

support two ways of thinking about division: Dividing means breaking the dividend into a certain number of equal parts, and when there are more parts, the size of each part gets smaller. Dividing means breaking the dividend into parts of a particular size, and when the size of each part gets larger, the number of parts gets smaller. To involve more students in the conversation, consider asking as the students share their ideas: “Who can restate ___’s reasoning in a different way?” “Did anyone solve the problem the same way but would explain it differently?” “Did anyone solve the problem in a different way?” “Does anyone want to add on to ___’s strategy?” “Do you agree or disagree? Why?”’ The teacher guidance does not gather information about students’ current understandings and proficiencies with responsiveness to language support. Strategies for differentiation are not evident based on the level of multi-language learners.

- Unit 6, Expressions and Equations, Lesson 6: Write Expressions Where Letters Stand for Numbers, Activity 6.,1 Algebra Talk: When is 6, Activity Synthesis, includes differentiation strategies and supports to be responsive to students’ current understandings in content of expressions. Student Task, “If x is 6, what is $x + 4$, $7 - x$, x^2, $\frac{1}{3}x$.” Teacher guidance, “Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking: “Who can restate ___’s reasoning in a different way?” “Did anyone have the same strategy but would explain it differently?” “Did anyone solve the problem in a different way?” “Does anyone want to add on to _____’s strategy?” “Do you agree or disagree? Why?” The teacher guidance does not gather information about students’ current understandings and proficiencies with responsiveness to language support. Strategies for differentiation are not evident based on the level of multi-language learners.

- Unit 7, Rational Numbers, Lesson 4: Ordering Rational Numbers, Activity 4.2, Ordering Rational Number Cards, Activity Synthesis, includes differentiation strategies and supports to be responsive to students’ current understandings in content of ordering rational numbers. “The purpose of the discussion is to solidify students’ understanding of the order of rational numbers. Select previously, students to share how they decided how to place numbers like $-\frac{9}{8}, \frac{9}{8}\frac{8}{3}$, and $-22\frac{1}{2}$. Here are some questions to consider: Which numbers were hardest to place and which were the least difficult? How does placing negative numbers compare to placing positive numbers? How did you use numbers you had already placed to reason about where to place new numbers?” The teacher guidance does not gather information about students’ current understandings and proficiencies with responsiveness to language support. Strategies for differentiation are not evident based on the level of multi-language learners.

Grade 7:

- Unit 1, Scale Drawings, Lesson 6: Scaling and Area, Activity 6.3, Area of Scaled Parallelograms and Triangles, Activity Synthesis, includes differentiation strategies and supports to be responsive to students’ current understandings in content with scale factors.

"Invite selected students to share their solutions. Then focus class discussion on two themes: how the values in the tables for the two shapes compare, and how students determined the scaled areas for the scale factors 5 and $\frac{3}{5}$. Ask questions such as: What did you notice when you compared your answers with another group that worked with the other figure? (When the scale factors are the same, the scaled areas are the same, though the bases and heights are different.) How did you find the scaled areas for scale factors of 5 and $\frac{3}{5}$? (By scaling the original base and height and multiplying the scaled measurements; by multiplying the original area by (scale factor)^2.) How is the process for finding scaled area here the same as and different than that in the previous activities with pattern blocks? (The area units are different; the pattern of squaring the scale factor is the same.) Highlight the connection between the two ways of finding scaled areas. Point out that when we multiply the base and height each by the scale factor and then multiply the results, we are essentially multiplying the original lengths by the scale factor two times. The effect of this process is the same as multiplying the original area by (scale factor)^2." The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Differentiation is not evident based on the level of multi-language learners.

- Unit 3, Measuring Circles, Lesson 3: Exploring Circumference, Activity 3.2, Measuring Circumference and Diameter, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of diameter and circumference. "As students work, monitor and select students who notice that the relationship between diameter and circumference appears to be proportional, and ask them to share during the whole-group discussion. If students are using the digital version of the activity, they don't necessarily need to measure physical objects, but we recommend they do so anyway." The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Differentiation is not evident based on the level of multi-language learners.

- Unit 7, Angles, Triangles, and Prisms, Lesson 12: Volume of Right Prisms, Activity 12.2, Finding Volume with Cubes, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of understanding the volume of right prisms. "Select students to share their reasoning. Consider asking some of the following questions: "How do you know this figure is a prism?" (Cross sections parallel to the base are identical copies.) "What is the area of the base of this figure?" (It is the number of cubes in one layer of the prism.) "How do you calculate the total number of cubes to make the prism?" (Multiply the number of cubes in one layer by the number of layers.) "What is the volume of this prism?" (The volume is the same as calculating the number of cubes to make the prism.) "If you find the area of the base, how do you use that information to calculate the volume of the prism?" (Multiply the area of the base by the height of the prism.) "How would the volume of the prism change if we changed the shape of the base but still used 27 cubes to build it?" (The volume would not change.) If not mentioned by

students, explain that calculating the total number of cubes to make the prism is the same as calculating the volume of the prism. We can find the area of the base of the prism and multiply that by the number of layers in the prism which is the same as the height of the prism. The height of the prism is measured in units, the area of the base is measured in $units^2$ and the volume of the prism is measured in $units^3$." The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Strategies for differentiation are not evident based on the level of multi-language learners.

- Unit 9, Putting it All Together, Lesson 3: More Costs of Running a Restaurant, Activity 3.2, Disposable or Reusable?, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of proportional relationships. "Invite a couple of students to share their answers and strategies for each problem. Record and display their explanations for all to see. Refer to MLR 2 (Collect and Display). After evaluating all four expressions, ask students: "What do you notice about the value of each expression as the divisor (the number we use to divide) gets larger?" "Why is the value of the expression getting smaller each time?" Highlight explanations that support two ways of thinking about division: Dividing means breaking the dividend into a certain number of equal parts, and when there are more parts, the size of each part gets smaller. Dividing means breaking the dividend into parts of a particular size, and when the size of each part gets larger, the number of parts gets smaller. To involve more students in the conversation, consider asking as the students share their ideas: "Who can restate ___'s reasoning in a different way?" "Did anyone solve the problem the same way but would explain it differently?" "Did anyone solve the problem in a different way?" "Does anyone want to add on to ___'s strategy?" "Do you agree or disagree? Why?" The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Strategies for differentiation are not evident based on the level of multi-language learners.

Grade 8:

- Unit 2, Dilations, Similarity, and Introducing Slope, Lesson 10: Meet Slope, Activity 10.4., Meet Slope, Different Slopes of Different Lines, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of finding slope using triangles. "Two important conclusions for students to understand are: Given a line on a grid, they can draw a right triangle whose longest side is on the line, and then use the quotient of the vertical and horizontal sides to find the slope. Given a slope, they can draw a right triangle using vertical and horizontal lengths corresponding to the slope, and then extend the longest side of the right triangle to create a line with that slope. When discussing line F, ask students to share how they drew their triangle. If possible, select students who drew their triangles correctly but at a different scale (for example, one student who used a triangle with a vertical length of 1 and a horizontal length of 2, and a different student who used a vertical length of 4 and a horizontal length of 8). Demonstrate (or get

students who have drawn different triangles to do so) that the *quotient* of side lengths is the important feature, since any triangle drawn to match a given slope will be similar to any other triangle drawn to match the same slope." The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Differentiation is not evident based on the level of multi-language learners.

- Unit 4 Linear Equations and Linear Systems, Lesson 3: Balanced Moves, Activity 3.2, Matching Equation Moves, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of balanced equations. "The goal of this discussion is to get students using the language of equations and describing the changes happening on each side when solving. Ask: "What is a move you could do to the equation $7 = 2x$ on card 1 that would result in an equation of the form $x = 7$? What is another move that would also work?" "Which numbered card was the most challenging to match?" "Does anyone have a value that would solve one of the numbered cards? How did you figure it out?" End the discussion by inviting groups to share the equations they wrote for card 6 and describe how they match the move "add $6x$ to each side." The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Differentiation is not evident based on the level of multi-language learners.

- Unit 5, Functions and Volume, Lesson 2: Introduction to Functions, Activity 2.3, Using Function Language, Activity Synthesis, includes differentiation strategies and supports to be responsive to students' current understandings in content of using function language. "The goal of this discussion is for students to use the language like "[the output] depends on [the input]" and "[the output] is a function of [the input]" while recognizing that a function means each input gives exactly one output. Begin the discussion by asking students if any of them had a different response from their partner that they were not able to reach agreement on. If any groups say yes, ask both partners to share their responses. Next, select groups to briefly share their responses for the other questions and address any questions. For example, students may have a correct answer but be unsure since they used different wording than the person who shared their answer verbally with the class. If time permits, give groups 1–2 minutes to invent a new question like the ones in the task that is not a function. Select 2–3 groups to share their question and ask a different group to explain why it is not a function using language like, "the input does not determine the output because. . . ." The teacher guidance is responsive to students' current understandings in content, however, does not gather information about students' current understandings and proficiencies with responsiveness to language support.

- Unit 8, Pythagorean Theorem and Irrational Numbers, Lesson 3: Algebra Talk: Positive Solutions, Activity 3.2, Three Squares, Access for English Learners, includes differentiation strategies and supports to be responsive to students' current understandings in content of Pythagorean Theorem. "As students share their answer for the area and side length of the tilted square, press for details in students' reasoning by asking how they know the area is 2

square units. Listen for and amplify the language students use to describe either the "decompose and rearrange" or the "surround and subtract" method for finding the area of the tilted square." The teacher guidance does not gather information about students' current understandings and proficiencies with responsiveness to language support. Differentiation is not evident based on the level of multi-language learners.

3b Report

Criterion 3	Materials provide responsive language and collaborative supports that amplify mathematical language development.
Indicator 3b	Materials amplify language and mathematics content while maintaining task and text complexity (oral and written).

While materials do amplify language and mathematics content while maintaining task and text complexity in both oral and written modes, there is a heavy presence of oral complexity. Examples include:

Grade 6:

- Unit 1, Areas and Surface, Lesson 12: What is Surface Area?, Activity 12.2, Covering the Cabinet (Part 2), Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Speaking, Listening: MLR7 Compare and Connect. As students share their strategies for determining the number of sticky notes that cover the cabinet, ask students to make connections between the various strategies. Some students will calculate the number of sticky notes that will cover each of the five faces of the cabinet and add them together. Other students may realize that opposite faces of the cabinet are congruent so it is only necessary to calculate the area of three faces of the cabinet. Encourage students to explain why both methods result in the same answer. This will promote students' use of mathematical language as they make sense of the various methods for finding the surface area of a rectangular prism. Design Principles(s): Cultivate conversation; Maximize meta-awareness." Students' use of mathematical language as they make sense of the various methods for finding the surface area of a rectangular prism.
- Unit 5, Arithmetic in Base Ten, Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits, Activity 4.3, Missing Numbers, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Speaking, Listening, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Display each of the five problems for all to see. For each response that is shared, ask students to restate what they heard using precise mathematical language. Annotate the display to illustrate the steps students describe, and label the strategy that was used next to

each question (for example, "work backwards," or "write additional zeros"). Invite students to suggest additional details to include on the display that will support their understanding of each approach." Students restate what they heard using precise mathematical language and suggest additional details.

- Unit 6, Expressions and Equations, Lesson 13: Expressions with Exponents, Activity 13.2, Is This Equation True, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Revoice language and push for clarity in reasoning when students discuss their strategies for determining whether the equations are true or false. Provide a sentence frame such as "The equation is true (or false) because ___." This will strengthen students' mathematical language use and reasoning when discussing the meaning of exponents and operations that can make the equivalence of expressions true or false." Students justify their reasoning when analyzing the equations, while providing them the language support to reach this goal.

Grade 7:

- Unit 3, Measuring Circles, Lesson 2: Exploring Circles, Activity 2.2, Sorting Round Objects, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Writing, Representing, Conversing: MLR5 Co-craft Questions. Use this routine to get students in the mindset of observing objects and to develop descriptive language to distinguish between objects that are circular and those that are not. Begin by showing only the blackline master, and do not present the activity yet. Ask students to jot down possible mathematical questions that could be asked about the images. This invites participation from all students and lowers the pressure for using specific math language yet. After a minute or two of think time, invite students to compare their questions with a partner. Conclude the routine by asking some pairs to share their questions aloud with the class before moving on to the original activity. Design Principle(s): Maximize meta-awareness; Cultivate conversation." Students develop descriptive language to distinguish between the differences of circular and noncircular.

- Unit 5, Rational Number Arithmetic, Lesson 4: Money and Debt, Activity 4.2, Cafeteria Food Debt, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Speaking, Listening: MLR8 Discussion Supports. Demonstrate the use of mathematical language to describe thinking. For example, "If Kiran begins the month with $24, I know that he starts with a positive amount. I know he spends $16, and spending money is subtraction or negative. To create an equation, I need both values." To provide an opportunity for both listening and speaking, ask students, "Who can restate my reasoning in a different way?" This helps invite more student participation and meta-awareness of language and reasoning." Students restate what they heard using precise mathematical language and suggest additional details.

- Unit 9, Putting it all Together, Expressions and Equations, Lesson 7: More Expressions and Equations, Activity 7.2, A Souvenir Stand, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "When students explain how they

wrote their expressions, provide sentence frames such as "___ represents ___ because . . . ." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. This will help students to explicitly connect the language of the problem with the structure of the expressions that represent the context." Students are provided a sentence frame to help them explicitly connect the language and the structure of expressions.

Grade 8:

- Unit 1, Rigid Transformations and Congruence, Lesson 7: No Bending or Stretching, Activity 7.2, Sides and Angles, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Speaking: MLR8 Discussion Supports. As students describe their approaches, press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example of their process. Connect the terms corresponding sides and corresponding angles to students' explanations multi-modally by using different types of sensory inputs, such as demonstrating the transformation or inviting students to do so, using the images, and using gestures. This will help students to produce and make sense of the language needed to communicate their own ideas." Students describe, explain, and provide examples to maintain task and text complexity.

- Unit 2 Dilations, Similarity, and Introducing Slope, Lesson 10: Meet Slope, Activity 10.1, Equal Quotients, Activity Synthesis, illustrates amplifying language while maintaining task and text complexity. The teacher is guided to ask students if they can think of some different ways to write numbers that are equal to 1 divided by 2. Then they are asked to come up with at least 3 to 4 different numbers equal to 15 divided by 2. Teachers state, "Ask students to share their strategies for finding numbers equal to. Record and display their answers and strategies for all to see. To involve more students in the conversation, consider asking:"Who can restate ___'s reasoning in a different way?" "Did anyone solve the problem the same way but would explain it differently?" "Did anyone solve the problem in a different way?" "Does anyone want to add on to ___'s strategy?" "Do you agree or disagree? Why?" Students restate what they heard using precise mathematical language and suggest additional details using sentence structures.

- Unit 8, Pythagorean Theorem and Irrational Numbers, Lesson 7: A Proof of the Pythagorean Theorem, Activity 7.3: Let's Take it for a Spin, Access for English Learners, illustrates amplifying language while maintaining task and text complexity. "Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their methods for finding the side length of a right triangle, present an incorrect solution based on a common error you observe in the class. For example, "I know that a =

- $\sqrt{8}$, b = 4 and c = y, so when I use the Pythagorean Theorem, I get the equation $(\sqrt{8})^2 + 4^2 = y^2$. This equation simplifies to 8 +16 = y^2. When I solve for y, I get $y = \sqrt{24}$." Ask students to identify the error, critique the reasoning, and revise the original statement. As students

discuss in partners, listen for students who clarify the meaning of each term in the equation $a^2 + b^2 = c^2$. In the Pythagorean Theorem, a^2 and b^2 represent the square of the legs of the right triangle, whereas c^2 represents the square of the hypotenuse. This routine will engage students in meta-awareness as they critique and correct a common error when applying the Pythagorean Theorem." Students critique the language used to discuss a specific mathematical concept and identify the language that highlights the goal of the lesson.

At the program level:

A Sentence Frames chart provides guidance for teachers and is exclusively located within the IM Curriculum. IM Curriculum, Access For English Language Learners, provides guidance for teachers to use sentence frames to amplify language and mathematics content while maintaining task and text complexity. "Sentence frames can support student language production by providing a structure to communicate about a topic. Helpful sentence frames are open-ended, so as to amplify language production, not constrain it. The table shows examples of generic sentence frames that can support common disciplinary language functions across a variety of content topics. Some of the lessons in these materials include suggestions of additional sentence frames that could support the specific content and language functions of that lesson." Examples include:

- "describe- It looks like…,I notice that…, I wonder if…, Let's try…, A quantity that varies is _____., What do you notice?, What other details are important?
- explain - First, I _____ because…, Then/Next, I…, I noticed _____ so I…, I tried _____ and what happened was…, How did you get…?, What else could we do?
- justify- I know _____ because…, I predict _____ because…, If _____ then _____ because…, Why did you…?, How do you know…?, Can you give an example?
- generalize - _____ reminds me of _____ because…, _____ will always _____ because…, _____ will never _____ because…, Is it always true that…?, Is _____ a special case?
- critique- That could/couldn't be true because…, This method works/doesn't work because…, We can agree that…, _____'s idea reminds me of…, Another strategy would be _____ because…, Is there another way to say/do...?
- compare and contrast- Both _____ and _____ are alike because…, _____ and _____ are different because…, One thing that is the same is…, One thing that is different is…, How are _____ and _____ different?, What do _____ and _____ have in common?
- represent - _____ represents _____., _____ stands for _____., _____ corresponds to _____., Another way to show _____ is…, How else could we show this?
- interpret - We are trying to…, We will need to know…, We already know…, It looks like _____ represents…, Another way to look at it is…, What does this part of _____ mean?, Where does _____ show...?"

3c Report

Criterion 3	Materials provide responsive language and collaborative supports that amplify mathematical language development.
Indicator 3c	Materials include use of language structures or routines that make full use of and engage all forms of communication including math conversations.

According to the IM Curriculum, Access For English Language Learners, Principle 3: CULTIVATE CONVERSATION, “Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning, communicate that meaning, and refine the way content understandings are communicated. When students have a purpose for talking and listening to each other, communication is more authentic. During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including when having conversations, making claims, justifying claims with evidence, making conjectures, communicating reasoning, critiquing the reasoning of others, engaging in other mathematical practices, and above all when making mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.” Examples include:

From Grades 6-8:

Access For English Learners are provided for teachers in each lesson. IM Curriculum, Access For English Language Learners, states, “Each lesson includes instructional strategies that teachers can use to facilitate access to the language demands of a lesson or activity. These support strategies, labeled “Access for English Learners," stem from the design principles and are aligned to the language domains of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). They provide students with access to the mathematics by supporting them with the language demands of a specific activity without reducing the mathematical demand of the task. Using these supports will help maintain student engagement in mathematical discourse and ensure that the struggle remains productive. Teachers should use their professional judgment about which routines to use and when, based on their knowledge of the individual needs of students in their classroom.”

Grade 6:

- Unit 2, Introducing Ratios, Lesson 9: Constant Speed, Activity 9.2, Moving 10 Meters Dividing by Powers of 10, Access for English Learners, illustrates the use of language structures and

routines that make full use of and engage all forms of communications. “Representing: MLR7 Compare and Connect. As students share different approaches for reasoning about distance traveled in 1 second, ask students to identify "What is the same and what is different?" about the approaches. Help students connect approaches by asking "Where do you see the measurement of speed '___ meters per second' in each approach?" Students communicate, listen, and read as they connect the concept of rate with a visual rate representation.

- Unit 4, Dividing Fractions, Lesson 4: How Many Groups? (Part 1), Activity 4.2, Reasoning with Pattern Blocks, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. “Representing, Writing: MLR3 Clarify, Critique, Correct. At the appropriate time, pause the class for a brief discussion of the first question. Display the following incorrect response that reflects a possible common misunderstanding: “The area of the rhombus is 3 because 3 fits inside the hexagon.” Ask students, “Do you agree with the statement? Why or why not?” Invite students to identify the error, correct the statement, and draw a diagram to represent the situation. Improved statements should include fractional language and direct connections to the diagram. This will help students evaluate and improve on the written mathematical arguments of others.” Students communicate, listen, read, and write as they make sense of incorrect solutions or strategies as they discuss and write their explanations.
- Unit 6, Expressions and Equations, Lesson 12: Meaning of Exponents, Activity 12.2, The Genie’s Offer, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. Students make connections as they discuss the idea of multiplying *n* factors of 2 to get the expression 2^n. “Speaking and Listening: MLR1 Stronger and Clearer Each Time. Use this routine to provide students with a structured opportunity to refine their explanations about whether or not the value of the magical coins will exceed a million dollars within the 28 days. Give students time to meet with 2–3 partners, to share and get feedback on the first draft of their response. Provide students with prompts they can use to give each other feedback (e.g., “Can you explain how . . .”, “You should expand on . . .” etc.). This will give students an opportunity to strengthen their ideas and clarify their language.” Students communicate, listen, and read make sense of incorrect solutions or strategies as they discuss and write their explanation.

Grade 7:

- Unit 2, Introducing Proportional Relationships, Lesson 9: Solving Problems about Proportional Relationships, Activity 9.3, Moderating Comments, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. “Representing, Conversing: MLR7 Compare and Connect. Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite students to create a visual display showing which job applicant should get the job and why. Allow students time to quietly circulate and analyze the selections and justifications in at least 2 other displays in the room. Give students quiet think time to consider what is the

same and what is different and whether or not they agree. Next, ask students to find a partner to discuss what they noticed. Listen for and amplify observations that highlight advantages and disadvantages to each method of determining the top job applicant. This will help students identify situations when comparing rates is more effective than the sum of quantities. Design Principle(s): Optimize output (for justification) Cultivate conversation." Students communicate, listen, and read as they explain and justify their reasoning as they explain their differences.

- Unit 7, Angles, Triangles, and Prisms, Lesson 12: Volume of Right Prisms, Activity 12.2, Finding Volume with Cubes, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. "Writing, Listening, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Invite students to draft an initial response to the question: "How do you know if a three-dimensional figure is a prism?" Give students time to meet with 2–3 partners, to share and get feedback on their responses. Provide students with prompts for feedback that will help their partner strengthen their ideas and clarify their language (e.g., "What are some properties of prisms?", "Can you give an example and non-example?", etc.). Students can borrow ideas and language from each partner to refine and clarify their explanation. This will also help students to build and describe the three-dimensional figures used in this lesson and the unit." Students communicate, listen, read, and write as they expand and clarify their responses as they connect the concept of rate to visual representations.

- Unit 9, Putting it All Together, Lesson 5: How Crowded Is this Neighborhood?, Activity 5.2, Dot Density with a Twist, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. "Writing, Conversing: MLR5 Co-craft Questions. Display only the image of the four dot arrays without revealing any of the questions that follow. Give students 1–2 minutes to write a list of possible mathematical questions they could ask about the arrays. Invite students to share their questions with a partner, and then select 2–3 students to share their questions with the whole class. Highlight any questions that refer to how dots are "distributed," even if students do not use that particular phrase. Finally, reveal the whole problem with text so that students may begin addressing the questions. This helps amplify language related to the distribution of dots." Students communicate, listen, read, and write questions and explain their reasoning with the class.

Grade 8:

- Unit 2, Introducing Ratios, Lesson 6: Similarity, Activity 6.2, Similarity Transformations (Part 1), Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. "Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have determined the sequence of transformations that shows the polygons are similar, ask students to write a detailed

sequence of the transformations on their paper. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “How did you know to translate point L to point E?”, and “How did you know to dilate the polygon by a scale factor of 3?”, etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to show polygons are similar using a sequence of transformations. Design Principles(s): Optimize output (for explanation) Maximize meta-awareness.” Students utilize speaking, listening, reading, and writing as they work with similarity in transformations.

- Unit 5, Functions and Values, Lesson 9: Linear Models, Activity 9.2, Shadows, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. “Representing, Conversing: MLR7 Compare and Connect. Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite students to create a visual display showing which job applicant should get the job and why. Allow students time to quietly circulate and analyze the selections and justifications in at least 2 other displays in the room. Give students quiet think time to consider what is the same and what is different and whether or not they agree. Next, ask students to find a partner to discuss what they noticed. Listen for and amplify observations that highlight advantages and disadvantages to each method of determining the top job applicant. This will help students identify situations when comparing rates is more effective than the sum of quantities. Design Principle(s): Optimize output (for justification) Cultivate conversation.” Students communicate, listen, and read as they compare and contrast linear models and justify their reasoning.
- Unit 9, Putting it All Together, Lesson 1: Tessellations of the Plane, Activity 1.3, Describing a Tessellation Launch, Access for English Learners, illustrates the use of language structures and routines that make full use of and engage all forms of communications. “MLR2 Speaking, Listening, Representing: Collect and Display. While students work with their partners during this activity, use this to record initial language used to describe their tessellations. Since students are not required to use specific language like rotation or translation yet, record the language students use as they construct and compare their tessellations. Display the recording language throughout the activity, and refer to it during the synthesis. Add or edit language as students respond to each question.” Students communicate, listen, and read exemplary language needed to meet the lesson goal and further mathematical concepts.

3d Report

Criterion 3	Materials provide responsive language and collaborative supports that amplify mathematical language development.
Indicator 3d	Materials include guidance for intentional and flexible grouping structures to ensure equitable participation.

Materials do provide guidance for flexible grouping structures, but do not intentionally ensure equitable participation. IM Curriculum, Access For English Language Learners, “Principle 3: CULTIVATE CONVERSATION Strengthen opportunities and structures for constructive mathematical conversations (pairs, groups, and whole class). Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning, communicate that meaning, and refine the way content understandings are communicated. When students have a purpose for talking and listening to each other, communication is more authentic. During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including when having conversations, making claims, justifying claims with evidence, making conjectures, communicating reasoning, critiquing the reasoning of others, engaging in other mathematical practices, and above all when making mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.” Lesson level examples include:

Grade 6:

- Unit 2, Introducing Ratios, Lesson 9: Constant Speed, Activity 9.1, Number Talk: Dividing by Powers of 10, Access for English Learners, provides minimal guidance for forming and structuring group work. “Speaking: MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class. Design Principle(s): Optimize output (for explanation).” This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups to ensure equitable participation.
- Unit 4, Dividing Fractions, Lesson 3: Interpreting Division Situations, Activity 3.2, Homemade Jams, Launch, provides minimal guidance for forming and structuring group work. “Arrange

students in groups of 2. Tell the class that you will read the three story problems, and ask them to be prepared to share at least one thing they notice and one thing they wonder. After reading, give them a minute to share their observations and questions with their partner. Clarify that their job is to draw a diagram and write a multiplication equation to express the relationship in each story and then answer the question. Give students 7–8 minutes of quiet work time, followed by 2–3 minutes to share their work with their partner." This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups to ensure equitable participation.

- Unit 6, Expressions and Equations, Lesson 2: Truth and Equations, Activity 2.3, Using Structure to Find Solutions, Launch, provides minimal guidance for forming and structuring group work. "Arrange students in groups of 2. Tell the class that you will read the three story problems, and ask them to be prepared to share at least one thing they notice and one thing they wonder. After reading, give them a minute to share their observations and questions with their partner. Clarify that their job is to draw a diagram and write a multiplication equation to express the relationship in each story and then answer the question. Give students 7–8 minutes of quiet work time, followed by 2–3 minutes to share their work with their partner." This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups to ensure equitable participation.

Grade 7:

- Unit 3, Measuring Circles, Lesson 3: Exploring Circumference, Activity 3.2, Measuring Circumference and Diameter, Launch, provides minimal guidance for forming and structuring group work. "Arrange students in groups of 2–4. Distribute 3 circular objects and measuring tapes or string and rulers to each group. Especially if using string and rulers, it may be necessary to demonstrate the method for measuring the circumference. Ask students to complete the first two questions in their group, and then gather additional information from two other groups (who measured different objects) for the third question." This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups to ensure equitable participation.
- Unit 4, Proportional Relationships and Percentages, Lesson 7: One Hundred Percent, Activity 7.1, Notice and Wonder: Double Number Line, Launch, provides minimal guidance for forming and structuring group work. In this activity students look at two number lines and think about what they notice and wonder. "Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion." This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups to ensure equitable participation.

- Unit 6, Expressions, Equations, and Inequalities, Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2), Activity 5.2, More Situations and Diagrams, Launch, provides minimal guidance for forming and structuring group work. “Arrange students in groups of 2. Give 5–10 minutes to work individually or with their partner, followed by a whole-class discussion.” This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups for equitable participation.

Grade 8:

- Unit 1, Rigid Transformations and Congruence, Lesson 7: No Bending or Stretching, Activity 7.3, Which One?, Launch, provides minimal guidance for forming and structuring group work. “Give students 4 minutes quiet work time, 2 minutes to discuss with partner” and gives further guidance on how to monitor students for language: “As students discuss their work with a partner, listen for and collect the language students use to describe each transformation. Record students’ words and phrases on a visual display.” This example provides a structure for group work, and guidance on monitoring language used in the activity, but does not provide guidance for forming groups or for monitoring groups for equitable participation.
- Unit 5, Functions and Volume, Lesson 5: More Graphs of Functions, Activity 5.1, Which One Doesn’t Belong: Graphs, Launch, provides minimal guidance for forming and structuring group work. In this activity students look at two number lines and think about what they notice and wonder. “Arrange students in groups of 2–4. Display the image of the four graphs for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. Follow with a whole-class discussion.” This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups for equitable participation.
- Unit 8, Pythagorean Theorem and Irrational Numbers, Lesson 8: Finding Unknown Side Lengths, Activity 8.1, Which One Doesn't Belong: Equations, Launch, provides minimal guidance for forming and structuring group work. “Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reason why a particular equation does not belong and together find at least one reason each question doesn’t belong.” This example provides a structure for group work, but does not provide guidance for forming groups or for monitoring groups for equitable participation.

4a Report

Criterion 4	Materials forefront, value, and use the assets of students, including their home language, experiences, and beliefs, in the teaching of mathematics
Indicator 4a	Materials activate and build on students' home and community mathematical practices, showing teachers how to elicit and affirm students' strengths and experiences and connect these to mathematics learning.

The materials show teachers how to elicit and affirm students' strengths and experiences and connect these to mathematics learning, however, there are missed opportunities in building on students' home and community mathematical practices. Lesson level examples include:

Grade 6:

- Unit 4, Dividing Fractions, Lesson 1: Size of Divisor and Size of Quotient, Activity 1.2, All Stacked up, Activity Narrative, illustrates how materials sometimes activate and build on students' home and community mathematical practices. "This lesson aims to give students a concrete setting for thinking about division. Students estimate how many of each given object are needed to make a stack of a given height. To do so, they use what they know about the sizes of familiar objects (boxes, bricks, notebooks, and coins) and their intuition that it takes more of a thinner object and fewer of a thicker object to reach the same height. Later, they will use this idea to think about division more generally. We often refer to certain objects (coins, books, etc.) as having a thickness rather than a height. Clarify that "thickness" and "height" refer to the same dimension in these examples. The images of the boxes and the bricks show stacks with more items at the base. Clarify that we are concerned only with a stack with one item per layer. As students discuss in groups, monitor for those who: Can explain clearly why the relationship between the height of the object being stacked and the height of a stack. Can explain clearly why the situation can be represented with a division expression." This example shows how the materials make connections to the real world, however this activity is general and does not explicitly ask students to make connections to their home and community mathematical practices.
- Unit 9, Putting it All Together, Lesson 1: Fermi Problems, Activity 1.1, Warm-Up: Ant Trek, Student Task Statement, Launch, illustrates how materials sometimes activate and build on students' home and community mathematical practices. "How long would it take an ant to run from Los Angeles to New York City? Arrange students in groups of 1–4. Provide access to four-function calculators. If students are doing their own research, provide access to internet-enabled devices. If conducting a gallery walk at the end, provide access to tools for

making a visual display. Before starting, ask students to come up with a guess about the answer and poll the class. Record all guesses. Ask students to brainstorm the information they need to answer the question. Give the information provided when students ask for it, or provide access to internet-enabled devices so that students can find the information they need. Vital information to have on hand includes: The distance between Los Angeles and New York City is about 3,944 km. An ant can run about 18 mm per second. Also of interest is the fact that most ants do not live long enough to complete this trip. Many ants live for only a couple months. If students realize this, ask them how many ant lifetimes it would take for an ant to make this journey." This example shows how the materials make connections to the real world, however this activity is general and does not explicitly ask students to make connections to their home and community mathematical practices.

Grade 7:

- Unit 5, Rational Number Arithmetic, Lesson 2: Changing Temperatures, Activity 2.2, Warmer and Colder, Launch, illustrates how materials sometimes activate and build on students' home and community mathematical practices. "Arrange students in groups of 2. Ask them, "If the temperature starts at 40 degrees and increases 10 degrees, what will the final temperature be?" Show them this number line: (image of a number line with intervals 0 to 50 marked at every 10 units a ray with endpoint at zero an arrow at 40 is marked a ray with endpoint at 40 and an arrow at 50) Explain how the diagram represents the situation, including the start temperature, the change, and the final temperature. Point out that in the table, this situation is represented by an equation where the initial temperature and change in temperature are added together to find the final temperature. Next, ask students to think about the change in the second row of the table. Give students 1 minute of quiet work time to draw the diagram that shows a decrease of 5 degrees and to think about how they can represent this with an addition equation. Have them discuss with a partner for 1 minute. Ask a few students to share what they think the addition equation should be. Be sure students agree on the correct addition equation before moving on. Tell students they will be answering similar questions, first by reasoning through the temperature change using whatever method makes sense, then drawing a diagram to show the temperature change, and finally, by writing an equation to represent the situation. Give students 4 minutes of quiet work time followed by partner and then whole group discussion." This example shows an opportunity for the materials to provide guidance for teachers to elicit and affirm students' strengths and experiences and guidance on how to incorporate those strengths and experiences into mathematics learning, however, this activity is general and does not explicitly ask students to make connections to their home and community mathematical practices.
- Unit 6, Expressions, Equations, and Inequalities, Lesson 18: Subtraction in Equivalent Expressions, Activity 18.1, Warm-Up: Number Talk Additive Inverses, Activity Synthesis, illustrates how materials sometimes activate and build on students' home and community mathematical practices. This number talk elicits strategies and understandings students have for adding and subtracting signed numbers. "Ask students to share their strategies for

each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking: “Who can restate ___’s reasoning in a different way?” “Did anyone have the same strategy but would explain it differently?” “Did anyone solve the problem in a different way?” “Does anyone want to add on to ___’s strategy?” “Do you agree or disagree? Why?” This example shows an opportunity for the materials to provide guidance for teachers to elicit and affirm students’ strengths and experiences and guidance on how to incorporate those strengths and experiences into mathematics learning, however, this activity is general and does not explicitly ask students to make connections to their home and community mathematical practices.

Grade 8:

- Unit 2, Dilations, Similarity, and Introducing Slope, Lesson 7: Similar Polygons, Activity 7.1, All, Some, None: Congruence and Similarity, Activity Synthesis, illustrates how materials sometimes activate and build on students’ home and community mathematical practices. Students examine statements deciding in each case whether the statement is always true, sometimes true, or never true. “1. If two figures are congruent, then they are similar. 2. If two figures are similar, then they are congruent. 3. If an angle is dilated with the center of dilation at its vertex, the angle measure may change. Discuss each statement one at a time with this structure: Poll the class on their answer choice and display the answers. If everyone agrees on one answer, ask a few students to share their reasoning, recording it for all to see. If there is disagreement on an answer, ask students with opposing answers to explain their reasoning to come to an agreement on an answer.” This example shows an opportunity for the materials to provide guidance for teachers to elicit and affirm students’ strengths and experiences and guidance on how to incorporate those strengths and experiences into mathematics learning, however, this activity is general and does not explicitly ask students to make connections to their home and community mathematical practices.
- Unit 8, Pythagorean Theorem and Irrational Numbers, Lesson 16: When Is the Same Size Not the Same Size?, Activity 16.1, Three Figures, Activity Synthesis, illustrates how materials sometimes activate and build on students’ home and community mathematical practices. “This lesson aims to give students a concrete setting for thinking about division. “Tell students that in photography, film, and some consumer electronics with a screen, the ratio of the two sides of a rectangle is often called its aspect ratio. In the rectangles in this activity, the aspect ratios are 5:1, 2:1, and 1:1. Demonstrate how the length of one side is a multiple of the other, on each rectangle. Students may be familiar with selecting an aspect ratio when taking or editing photos. Some common aspect ratios for photos are 1:1,4:3 , and 16:9 . Also, from ordering school pictures, 5 by 7 and 8 by 10 may be common sizes they've heard of.” This example shows how the materials make connections to the real world, however this activity is general and does not explicitly ask students to make connections to their home and community mathematical practices.

4b Report

Criterion 4	Materials forefront, value, and use the assets of students, including their home language, experiences, and beliefs, in the teaching of mathematics
Indicator 4b	Materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics and to express their culture and identity.

While teachers are not always explicitly guided to have students do so, there are opportunities within the program level and lesson level for students to use home language and practices as resources for learning mathematics and expressing their culture and identity. Illustrative Mathematics provides numerous resources translated into Spanish and consistently provides guidance for teachers to encourage students to use their own language. Additionally, there are program-level resources that include guidance for incorporating student assets into classroom instruction.

Program-level supports for home language:

- IM Curriculum, Design Principles, The Five Practices, provide opportunities for students to use their own language, however, the curriculum never explicitly describes home language. “Selected activities are structured using Five Practices for Orchestrating Productive Mathematical Discussions (Smith & Stein, 2011): Introduce a new concept and associated language. Activities that introduce a new concept and associated language build on what students already know and ask them to notice or put words to something new. Formalize a definition of a term for an idea previously encountered informally. Activities that formalize a definition take a concept that students have already encountered through examples, and give it a more general definition.”
- IM Curriculum, What is a "Problem-Based" Curriculum?, Principles for Mathematics Teaching and Learning, “Teachers should build on what students know: New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them. In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students’ learning.”
- IM Curriculum, Access For English Language Learners, Mathematical Language Routines, guide teachers to create opportunities for students in which they could use home language and practices, although teachers are not directly guided to do so. “The mathematical

language routines were selected because they are effective and practical for simultaneously learning mathematical practices, content, and language. A mathematical language routine is a structured but adaptable format for amplifying, assessing, and developing students' language. The routines emphasize uses of language that is meaningful and purposeful, rather than about just getting answers. These routines can be adapted and incorporated across lessons in each unit to fit the mathematical work wherever there are productive opportunities to support students in using and improving their English and disciplinary language use. These routines facilitate attention to student language in ways that support in-the-moment teacher-, peer-, and self-assessment. The feedback enabled by these routines will help students revise and refine not only the way they organize and communicate their own ideas, but also ask questions to clarify their understanding of others' ideas."

Lesson-level support for home language:

Grade 6:

- Unit 1, Area and Surface Area, Lesson 11: Polygons, Activity 11.2, What Are Polygons?, Launch, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. "Arrange students in groups of 2–4. Give students 3–4 minutes of quiet think time. Afterwards, ask them to share their responses with their group and complete the second question together. If there is a disagreement about whether a figure is a polygon, ask them to discuss each point of view and try to come to an agreement. Follow with a whole-class discussion." Access for English Learners, "Conversing, Representing, Writing: MLR2 Collect and Display. As students work, listen for, collect, and display terms and phrases students use to describe key characteristics of polygons (e.g., polygon, edge, vertices). Remind students to borrow language from the display as needed. This will help students use mathematical language when describing polygons. Design Principle(s): Support sense-making; Maximize meta-awareness." This example explicitly calls out opportunities for students to use home language and practices as resources for learning mathematics, however, not explicitly guiding teachers how to use their home language to express their culture and identity.
- Unit 5, Arithmetic in Base Ten, Lesson 5: Decimal Points in Product, Activity 5.2, Fractionally Speaking Power of Ten, Launch, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. "Arrange students in groups of 2. Ask one student in each group to complete the questions for Partner A, and have the other take the questions for Partner B. Then ask them to discuss their responses, answer the second question together, and pause for a brief class discussion." Access for English Learners, "Representing: MLR2 Collect and Display. Use this routine while students are working through the first two questions. As students work, circulate and listen for the connections students make between the problems. Write the students' words and phrases on a visual display and update it throughout the remainder of the lesson. Listen for language like "the same," reciprocal, and inverse operation. Remind

students to borrow language from the display as needed." This example explicitly calls out opportunities for students to use home language and practices as resources for learning mathematics, however, not explicitly guiding teachers how to use their home language to express their culture and identity.

- Unit 7, Rational Numbers, Lesson 12: Constructing the Coordinate Plane, Activity 12.2, Axes Drawing Decisions, Access for English Learners, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. "Listening, Speaking: MLR2 Collect and Display. Listen for and display vocabulary and phrases students use to justify their choice of axes (e.g., "minimum/maximum - or -coordinate" or "appropriate units"). Continue to update collected language students used to explain their reasoning to their peers. Remind students to borrow language from the display during paired and whole-class discussions. Design Principle(s): Maximize meta-awareness." This activity highlights a routine that values and honors the student language used in the classroom, and moves toward more academic language, although it does not call out home language explicitly.

Grade 7:

- Unit 2, Introducing Proportional Relationships, Lesson 8: Comparing Relationships with Equations, Activity 8.1, Notice and Wonder: Patterns with Rectangles, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. "Arrange students in groups of 2. Tell students that they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something and to think about the additional questions. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion." This provides opportunities for students to use home language and practices as resources for learning mathematics, although teachers are not directly guided to do so.

- Unit 4, Proportional Relationships and Percentages, Lesson 6: Increasing and Decreasing, 6.2, More Cereal and a Discount Shirt, Access for English Learners, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. "Writing: MLR3 Clarify, Critique, Correct. Before students share their reasoning for each problem, present an incomplete strategy for finding the discount price for the shirt. For example, "The discount is 20%, so the price of the shirt is $3.70." Ask students to critique the reasoning, and work with a partner to write an improved explanation. Listen for and amplify the language students use to make sense of what is displayed (e.g., "I think they ___ because ___."), as well as the mathematical language that students use that strengthen their explanations. This will support student understanding of mathematical language related percent increase and percent decrease. Design Principle(s): Maximize meta-awareness." This activity highlights a routine that values

and honors the student language used in the classroom, and moves toward more academic language, although it does not call out home language explicitly.

- Unit 8, Probability and Sampling, Lesson 7: Simulating Multi-step experiments, Activity 7.1, Notice and Wonder: Ski Business, Launch, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. “Arrange students in groups of 2. Tell students that they will look at two images, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the images for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.” This example provides opportunities for students to use home language, but doesn’t guide teachers to encourage students to do so.

Grade 8:

- Unit 7, Exponents and Scientific Notation, Lesson 6: What about Other Bases?, Activity 6.2, What Happens with Zero and Negative Exponents?, Launch, illustrates how materials explicitly guide teachers to create opportunities for students to use home language and practices as resources for learning mathematics. “Arrange students in groups of 2. Tell students to work on their own and then share their reasoning with their partner after each has had a chance to complete the problem. Encourage students to look for similarities to work they have already done with base 10. Give students 10 minutes of work time, followed by a whole class discussion.” Access for English Learners, “Conversing, Representing, Writing: MLR2 Collect and Display. Use this routine to support small-group and whole-class discussion. Circulate and listen to students as they work, capture the vocabulary and phrases students use to describe the patterns they noticed and how these are related to exponent rules. Listen for students who make connections between repeated multiplication, reciprocals, and negative and positive exponents. Record their language and any relevant diagrams onto a visual display that can be referenced in future discussions. This will help students produce and make sense of the language needed to communicate their reasoning of exponent rules for powers of positive bases other than 10. Design Principle(s): Support sense-making; Maximize meta-awareness.” This example explicitly calls out opportunities for students to use home language and practices as resources for learning mathematics, however, not explicitly guiding teachers how to use their home language to express their culture and identity.

- Unit 8, Pythagorean Theorem and Rational Numbers, Lesson 1: The Areas of Squares and Their Side Lengths, Activity 1.2, Decomposing to Find Area, Access for English Learners, illustrates how the materials include opportunities for students to use home language and practices as resources for learning mathematics, but do not explicitly guide teachers to encourage their use. “Speaking, Listening: MLR7 Compare and Connect. As students prepare a visual display of their work for the last problem, look for students with different methods for calculating the area of the shaded square. As students investigate each other’s work, ask

them to share what worked well in a particular approach. During this discussion, listen for and amplify the language students use to describe the decomposing, rearranging, and subtracting strategy for finding the area. Then encourage students to make connections between the quantities used to calculate the area of the shaded square and the shapes in the diagram. For example, the quantity 100 represents the area, in square units, of the large square. The quantity 10.5 represents the area, in square units, of one of the triangles. This will foster students' meta-awareness and support constructive conversations as they compare strategies for calculating areas and make connections between quantities and the areas they represent. Design Principles(s): Cultivate conversation Maximize meta-awareness." This activity highlights a routine that values and honors the student language used in the classroom and moves toward more academic language, although it does not call out home language explicitly.

4c Report

Criterion 4	Materials forefront, value, and use the assets of students, including their home language, experiences, and beliefs, in the teaching of mathematics
Indicator 4c	Materials guide teachers to establish and maintain a classroom culture that encourages student participation and agency for language development.

According to the IM Curriculum, Course Guide, the curriculum includes ten instructional routines in addition to the eight Math Language Routines (MLRs) that guide teachers to establish and maintain a classroom culture that encourages participation and agency for language development. Instructional Routines and MLRs are used in every lesson, throughout instructional materials. Discussion and student centered activities engage student participation and agency.

Examples from the program-level:

- IM Curriculum, Course Guide, Instructional Routines, Group Presentations, "Some activities instruct students to work in small groups to solve a problem with mathematical modeling, invent a new problem, design something, or organize and display data, and then create a visual display of their work. Teachers need to help groups organize their work so that others can follow it, and then facilitate different groups' presentation of work to the class. Teachers can develop specific questioning skills to help more students make connections and walk away from these experiences with desired mathematical learning." Think Pair Share routine, "Students have quiet time to think about a problem and work on it individually, and then time to share their response or their progress with a partner. Once these partner

conversations have taken place, some students are selected to share their thoughts with the class."

- IM Curriculum, What is a Problem Based Curriculum?, Critical Practices, "Establishing norms: Norms around doing math together and sharing understandings play an important role in the success of a problem-based curriculum. For example, students must feel safe taking risks, listen to each other, disagree respectfully, and honor equal air time when working together in groups. Establishing norms helps teachers cultivate a community of learners where making thinking visible is both expected and valued."
- IM Curriculum, Access For English Language Learners, Theory of Action, "Students are agents in their own mathematical and linguistic sense-making. Mathematical language proficiency is developed through the process of actively exploring and learning mathematics. Language is action: in the very doing of math, students have naturally occurring opportunities to need, learn, and notice mathematical ways of making sense and talking about ideas and the world. These experiences support learners in using and expanding their existing language toolkits."

Examples from the lesson-level:

Grade 6:

- Unit 2, Introducing Ratios, Lesson 1: Introducing Ratios and Ratio Language, Activity 1.3, The Student's Collection, Activity, guides teachers to engage students in an activity that encourages participation and agency. "In this activity, students write ratios to describe objects in their own collection. They create a display of objects and circulate to look at their classmates' work. Students see that there are several ways to write ratios to describe the same situation." This evidence shows how students participate in creating visual displays of objects and sharing them with the class.
- Unit 3, Unit Rates and Percentages, Lesson 5: Comparing Speeds and Prices, Activity 5.2, More Treadmills, Access for English Learners, guides teachers to engage students in an activity that encourages participation and agency. "Speaking, Writing: MLR5 Co-Craft Questions. Display the constant speed of Tyler, Kiran, and Mai and ask pairs of students to write possible mathematical questions about the situation. They can also ask questions about information that might be missing, or even about assumptions that they think are important. Then, invite select pairs to share their questions with the class. Look for questions that require students to make comparisons about different speeds. Finally, reveal the actual questions students are expected to work on, and students are set to work. This routine creates space for students to produce the language of mathematical questions as well as develop the language used to talk about constant speed. Design Principle(s): Optimize output (for questioning) Cultivate conversation." This allows for students to engage with the material organically and creates agency in students to craft their own

questions. Additionally, it sets up a routine where all answers are valued, and there is no one right response, creating a strong and safe culture.

Grade 7:

- Unit 4, Proportional Relationships and Percentages, Lesson 16: Posing Percentage Problems, Activity 16.3, Displaying the News, Activity, guides teachers to engage students in an activity that encourages participation and agency. “In this activity, students work in groups and make a poster in their groups using one of their news items. Next, students go on a gallery walk and use sticky notes to ask questions about the information presented on each poster. They practice critiquing the reasoning of others as they study information they have not themselves worked on. They then go back and study the feedback they received from their classmates and revise their own work.” This evidence shows how students participate in creating a poster and critique each other’s reasoning which is only possible when classroom culture has been established that encourages student participation and agency.
- Unit 8, Probability and Sampling, Lesson 19: Comparing Populations with Friends, Activity 19.2, Info Gap: Comparing Populations, Activity, guides teachers to engage students in an activity that encourages participation and agency. “In this info gap activity, students work together to compare two populations from information about samples from each of the populations. Students must pay attention to the information they need in order to solve the problem and the types of questions they could ask to get to the answer. The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).” Students are given the agency to choose their own questions and decide what information they need to to solve the problems, including the language that they need to develop in order to best gather the information they need. The crafting of questions is student led, and encourages participation and agency in language development.

Grade 8:

- Unit 4, Linear Equations and Linear Systems, Lesson 8: How Many Solutions?, Activity 8.3, Make Use of Structure, Access for English Learners, guides teachers to engage students in an activity that encourages participation and agency. “Writing, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine for students to respond in writing to one of the three questions for whole-class discussion. Divide the class into thirds and assign each group of students one of the questions. Give students 3 minutes of quiet time to write a response. Invite students to meet with at least 2 other students to share and get feedback on their writing. Students should first meet with a partner that responded to the same question they did, before meeting with a student from a different group. Encourage listeners to ask clarifying questions such as, “Can you describe that using a different example?” or “What is another feature of that type of equation?” Invite the students to write a final draft based on

their peer feedback. This will help students solidify their understanding of the number of solutions in a given equation by conversing with their partners. Design Principle(s): Optimize output Cultivate conversation." This activity engages students in providing feedback to each other, giving each student the agency to provide the feedback that they think is needed for more, clear, math language. A class culture of feedback and growth is established and students are engaged in creating precise mathematical language.

- Unit 6, Associations in Data, Lesson 1: Organizing Data, Activity 1.3, Tables and Their Scatter Plots, Access for English Learners, guides teachers to engage students in an activity that encourages participation and agency. "Conversing, Writing: MLR5 Co-Craft Questions. Display only the four graphs without revealing the questions that follow. Invite pairs of students to write mathematical questions about the graph. Then, invite 2–3 groups to share their questions with the class. Look for questions that ask students to make sense of the relationship between the two quantities represented on each axis. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the relationship between quantities that are represented graphically. Design Principle(s): Maximize meta-awareness Support sense-making." Students are given the agency to choose their own questions and decide what information they need to to solve the problems, including the language that they need to develop in order to best gather the information they need. The crafting of questions is student led, and encourages participation and agency in language development.

5a Report

Criterion 5	Materials provide opportunities to consistently assess, analyze, and communicate progress while students have opportunities to incorporate feedback.
Indicator 5a	Materials include a formative assessment plan for language alongside content that includes a connection to established unit/lesson language goals.

Mathematical goals are always assessed, but language goals are not always specifically mentioned. The Cool-Downs align to the language goals/learning goals for the majority of lessons. When Cool-Downs do not ask students to explain within a specific lesson, the materials will ask students to use this language to explore the same concepts and are purposeful. Supports are found within program-level resources and lessons through learning goals and Cool-Downs.

Examples from the program-level:

- IM Curriculum, Summative Assessments, Design Principles for Summative Assessments states, “Students should get the correct answer on assessment problems for the right reasons, and get incorrect answers for the right reasons. To help with this, our assessment problems are targeted and short, use consistent, positive wording, and have clear, undebatable correct responses. When possible, extended response problems provide multiple ways for students to demonstrate understanding of the content being assessed, through some combination of arithmetic or algebra, use of representations (tables, graphs, diagrams, expressions, and equations) and explanation.”
- IM Curriculum, Cool-Downs is a consistent formative assessment and found within each lesson. “Each lesson includes a Cool-Down (also known as an exit slip or exit ticket) to be given to students at the end of the lesson. This activity serves as a brief checkpoint to determine whether students understood the main concepts of that lesson. Teachers can use this as a formative assessment to plan further instruction. What if the feedback from a Cool-Down suggests students haven’t understood a key concept? Choose one or more of these strategies: Look at the next few lessons to see if students have more opportunities to engage with the same topic. If so, plan to focus on the topic in the context of the new activities. During the next lesson, display the work of a few students on that Cool-Down. Anonymize their names, but show some correct and incorrect work. Ask the class to observe some things each student did well and could have done better. Give each student brief, written feedback on their Cool-Down that asks a question that nudges them to re-examine their work. Ask students to revise and resubmit. Look for practice problems that are similar to, or involve the same topic as the Cool-Down, then assign those problems over the next few lessons. Here is an example. For a lesson in grade 6, unit 2, the learning goals are: Understand that doubling, tripling, or halving a recipe yields something that tastes the same. Understand that “doubling, tripling, or halving the recipe” means “doubling, tripling, or halving each ingredient.” The Cool-Down reads: Usually when Elena makes bird food, she mixes 9 cups of seeds with 6 tablespoons of maple syrup. However, today she is short on ingredients. Think of a recipe that would yield a smaller batch of bird food but still taste the same. Explain or show your reasoning. A number of students responded with 8 cups of seeds and 5 tablespoons of maple syrup, and did not provide an explanation or show their reasoning. Here are some possible strategies: Notice that this lesson is the first of several that familiarize students with contexts where equivalent ratios carry physical meaning, for example, the taste of a recipe or the result of mixing paint colors. Over the next several lessons, there are more opportunities to reason about multiple batches of a recipe. When launching these activities, pause to assist students to interpret this correctly. Highlight the strategies of any students who use a discrete diagram or other representation to correctly represent multiple batches. Select the work of one student who answered correctly and one student whose work had the common error. In the next class, display these together for all to see (hide the students’ names). Ask each student to decide which interpretation is correct, and defend their choice to their partner. Select students to share their reasoning with the class who have different ways of representing that 9:6 is equivalent to 3:2,6:4, or 4 1/2 : 3. Write feedback for each student along the lines of "If this recipe is 3 batches, how

could you make 1 batch?" Allow students to revise and resubmit their work. Look for practice problems in upcoming lessons that require students to generate examples of different numbers of batches equivalent to a given ratio, and be sure to assign those problems." This Cool-Down assesses the mathematical goal, however the language goal is never specifically mentioned.

Examples from the lesson-level:

Grade 6:

- Unit 1, Area and Surface Area, Lesson 13: Polyhedra, Learning Goals, provides the learning and language targets, "Compare and contrast (orally and in writing) features of prisms and pyramids. and Comprehend and use the words "face", "edge", "vertex", and "base" to describe polyhedra (in spoken and written language)." Activity 13.4, Cool-Down - Three-Dimensional Shapes, states "Write your best definition or description of a polyhedron. If possible, use the terms you learned in this lesson. Which of these five polyhedra are prisms? Which are pyramids?" The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "write your best definition" and the learning goal asks students to use the desired vocabulary to describe polyhedron.
- Unit 2, Introducing Ratios, Lesson 3: Recipes, Learning Goals, provides the learning and language targets, "Draw and label a discrete diagram with circled groups to represent multiple batches of a recipe. Explain equivalent ratios (orally and in writing) in terms of different sized batches of the same recipe having the same taste. Understand that doubling or tripling a recipe involves multiplying the amount of each ingredient by the same number, yielding something that tastes the same." Activity 3.4 Cool-Down - A Smaller Batch of Bird Food states, "Usually when Elena makes bird food, she mixes 9 cups of seeds with 6 tablespoons of maple syrup. However, today she is short on ingredients. Think of a recipe that would yield a smaller batch of bird food but still taste the same. Explain or show your reasoning." The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "explain or show your reasoning," and the learning goal asks students to use the desired vocabulary to describe equivalent ratios.
- Unit 4, Dividing Fractions, Lesson 3: Interpreting Division Situations, Learning Goals, provides the learning and language targets, "Create an equation and a diagram to represent a multiplication or division situation involving fractions, and coordinate these representations (orally). Explain (using words and other representations) how to find the unknown quantity in a multiplication or division situation involving fractions. Interpret a verbal description of a multiplication situation (in spoken or written language), and identify which quantity is unknown, i.e., the number of groups, the amount in one group, or the total amount." Activity 3.4 Cool-Down - Rice and Beans states, "1. Here are three problems. Select all problems that can be solved using division. a. Jada cut 4 pieces of ribbon that were equal in length. She used a total of 5 feet of ribbon. How long, in feet, was each piece

of ribbon she cut? b. A chef bought 3 bags of beans. Each bag contains $1\frac{2}{5}$ kilograms of beans. How many kilograms of beans did she buy? c. A printer takes $2\frac{1}{2}$ seconds to print a flyer. It took 75 seconds to print a batch of flyers without stopping. How many flyers were in the batch? 2. Consider the problem: Andre poured 27 ounces of rice into 6 containers. If all containers have the same amount of rice, how many ounces are in each container? a. Write an equation to represent the situation. Use a "?" to represent the unknown quantity. b. Find the unknown quantity. Show your reasoning." The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "show your reasoning," and the learning goal asks students to explain how to divide fractions using words and other representations.

Grade 7:

- Unit 1, Scale Drawings, Lesson 7: Scale Drawings, Learning Goals, provides the learning and language targets, "Describe (orally) what a "scale drawing" is. Explain (orally and in writing) how to use scales and scale drawings to calculate actual and scaled distances. Interpret the "scale" of a scale drawing." Activity 7.4 Cool-Down - Length of a Bus and Width of a Lake states, "1. A scale drawing of a school bus has a scale of
- $\frac{1}{2}$ inch to 5 feet. If the length of the school bus is $4\frac{1}{2}$ inches on the scale drawing, what is the actual length of the bus? Explain or show your reasoning. 2. A scale drawing of a lake has a scale of 1 cm to 80 m. If the actual width of the lake is 1,000 m, what is the width of the lake on the scale drawing?" The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "Explain or show your reasoning" and the learning goal asks students to "explain (orally and in writing) how to use scales and scale drawings to calculate actual and scaled distances."
- Unit 3, Measuring Circles, Lesson 6: Estimating Area, Learning Goals, provides the learning and language targets, "Estimate the area of a complex, real-world region, e.g., a state or province, by approximating it with an irregular polygon, and indicate that it is an approximation when expressing the answer (orally and in writing). Explain (orally and in writing) how to calculate the area of an irregular polygon by decomposing it. Interpret floor plans and maps in order to identify the information needed to calculate area." Activity 6.4, Cool-Down - The Area of Alberta states, "Estimate the area of Alberta in square miles. Show your reasoning." The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "estimate and show your reasoning," and the learning goal asks students to explain their calculations.
- Unit 6, Expressions, Equations, and Inequalities, Lesson 21: Combining Like Terms (Part 2), Learning Goals, provides the learning and language targets, "Critique (in writing) methods for generating equivalent expressions with fewer terms. Generate expressions that are not equivalent, but differ only in the placement of parentheses, and justify (orally) that they are not equivalent. Write expressions with fewer terms that are equivalent to a given expression

that includes negative coefficients and parentheses." Activity 21.4, Cool-Down - How Many Are Equivalent? states, "Select all the expressions that are equivalent to 16*x* - 12 - 24*x* + 4. Explain or show your reasoning. 1. 4 + 16x - 12(1 + 2x). 2. 40*x* - 16. 3. 16*x* - 24*x* - 4 + 12. 4. -8*x* - 8. 4. 10(1.6*x* - 1.2 - 2.4*x* + 4)." The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "Explain or show your reasoning," and the learning goal asks students to explain or show how expressions are equivalent.

Grade 8:

- Unit 4, Linear Equations and Linear Systems, Lesson 7: All, Some or No Solutions, Learning Goals, provides the learning and language targets, "Compare and contrast (orally and in writing) equations that have no solutions or infinitely many solutions. Create linear equations in one variable that have either no solutions or infinitely many solutions, using structure, and explain (orally) the solution method." Activity 7.4 Cool-Down - Choose Your Own Solution states, "3*x* +8 = 3*x* + . What value could you write in after 3*x* that would make the equation true for: 1. no values of *x*? 2. all values of *x*? 3. just one value of *x*?" The formative assessment is developed alongside the content and is connected to the established lesson language goals. The Cool-Down includes comparing and contrasting values and explaining their solutions, and the learning goal asks students to "compare and contrast (orally and in writing) and explain (orally)" how to find solutions to equations.
- Unit 5, Functions and Volume, Lesson 5: More Graphs of Functions, Learning Goals, provides the learning and language targets, "Describe (orally and in writing) a graph of a function as "increasing" or "decreasing" over an interval, and explain (orally) the reasoning. Interpret (orally and in writing) a graph of temperature as a function of time, using language such as "input" and "output." Activity 5.4 Cool-Down - Diego's 10K Race states, "Diego runs a 10 kilometer race and keeps track of his speed. 1. What was Diego's speed at the 5 kilometer mark in the race? 2. According to the graph, where was Diego when he was going the slowest during the race? 3. Describe what happened to Diego's speed in the second half of the race (from 5 km to 10 km)." The formative assessment is developed alongside the content and connected to the established lesson language goals. The Cool-Down includes "Describe what happened to Diego's speed," and the learning goal asks students to describe a graph of a function over an interval as increasing or decreasing.
- Unit 7, Exponents and Scientific Notation, Lesson 5: Negative Exponents with Powers of 10, Learning Goals, provides the learning and language targets, "Describe (orally and in writing) how exponent rules extend to expressions involving negative exponents. Describe patterns in repeated multiplication and division with 10 and
- $\frac{1}{10}$, and justify (orally and in writing) that $10^{-n} = \frac{1}{10^n}$." Activity 5.4 Cool-Down - Negative Exponent True or False states, "Mark each of the following equations as true or false. Explain or show your reasoning. 1. 10^{-5} = -10^5. 2.$(10^2)^{-3} = (10^{-2})^3$. 3. $\frac{10^3}{10^{14}} = 10^{-11}$." The formative

assessment is developed alongside the content and is connected to the established lesson language goals. The Cool-Down includes "Explain or show your reasoning," and the learning goal asks students to describe exponent patterns.

5b Report

Criterion 5	Materials provide opportunities to consistently assess, analyze, and communicate progress while students have opportunities to incorporate feedback.
Indicator 5b	Materials include guidance for gathering, analyzing, using, and communicating language and content data from formative assessments (in a cycle of continuous improvement).

According to the IM Curriculum, Cool-Downs guide teachers to respond to the feedback they collect within the lesson. "What if the feedback from a Cool-Down suggests students haven't understood a key concept? Choose one or more of these strategies: Look at the next few lessons to see if students have more opportunities to engage with the same topic. If so, plan to focus on the topic in the context of the new activities. During the next lesson, display the work of a few students on that Cool-Down. Anonymize their names, but show some correct and incorrect work. Ask the class to observe some things each student did well and could have done better. Give each student brief, written feedback on their Cool-Down that asks a question that nudges them to re-examine their work. Ask students to revise and resubmit. Look for practice problems that are similar to, or involve the same topic as the Cool-Down, then assign those problems over the next few lessons." Guidance for gathering, analyzing, using, and communicating language and content data from formative assessments (in a cycle of continuous improvement) is found within Lesson Cool-Downs, Cool-Downs Anticipated Misconceptions, and Activity Anticipated Misconceptions and Activity Synthesis.

Examples from the lesson-level:

Grade 6:

- Unit 2, Introducing Ratios, Lesson 13: Tables and Double Number Line Diagrams, Activity 13.4, Cool-Down - Bicycle Sprint, Responding to Student Thinking, provides guidance for teachers responding to student answers and provides feedback focusing on the lessons most important activities. "2. Points to emphasize, If students struggle with comparing values in double number lines in the Cool-Down, plan to revisit comparing tables and values when opportunities arise over the next several lessons. For example, in the first three

practice problems in Lesson 13, continue to compare and interpret tables by finding unit rates."

- Unit 3, Unit Rates and Percentages, Lesson 3: Measuring with Different-Sized Units, Activity 3.2, Measurement Stations, Anticipated Misconceptions, provides evidence around gathering, analyzing, using, and communicating language and content data from the stations activity. Students are investigating the idea of using different units to measure the same set of items. "At Station 1, students may count the number of base-10 centimeter rods rather than the number of centimeter cubes. Remind them that the question prompts for the number of cubes. At Station 2, students may need reminders about measuring objects at the zero marking on the ruler and about keeping the ruler going straight, both of which will affect the answer. Show them they can measure along the edge of the object to make sure the ruler is not veering off in one direction or another. At Station 4, students may be unclear about how to change the output unit on the scale for each object. Consider showing the class ahead of time. Students who are able to distinguish between weight and mass might say they cannot accurately compare their measurements. Clarify that we are talking only about the weight of the objects on Earth's surface. At Station 5, some students may consistently use under-filled or rounded teaspoons of salt, so their data will not reveal the 5 : 1 ratio of milliliters to teaspoons. Repeat the demonstration of how to measure a level teaspoon for them. Students may answer 3 milliliters for the question about 15 teaspoons because they divided by 5 instead of multiplying by 5. Encourage them to pay attention to which unit is bigger and ask what that tells them about which numerical value should be larger." Activity Synthesis, "Though much of the discussion will take place within groups, spend a few minutes ensuring that everyone understands the answers to the five questions. To conclude the activity, invite students to share anything that surprised them from the measuring work."
- Unit 8, Data Sets and Distribution, Lesson 8: Describing Distribution on Histograms, Activity 8.1, Which One Doesn't Belong: Histograms, encourages students to make sense of histograms in terms of center and spread while engaging in mathematical conversations and explaining their reasoning. Launch, "Arrange students in groups of 2–4. Display the images for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed one image that does not belong and can explain why. When the minute is up, give students 2 minutes to share their thinking with their small group, and then, together, find a reason that each image doesn't belong." Student Response, "Answers vary. Sample responses: Histogram B does not belong. Unlike the others, its distribution is not centered around 100. Histogram C does not belong. The spread of the data is much wider than that of the other histograms. Histogram D does not belong. It represents a smaller set of data compared to the others." Teachers provide guidance on how to respond to the data through Activity Synthesis. "Ask students to share one reason their group decided a particular image does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given

are reasonable. If students use terms that are essential in this unit (such as center, spread, distribution, frequency, etc.), ask them to explain their meanings in their own words; these are opportunities to reinforce their understanding of the terms and to note any misconceptions. If students give unsubstantiated claims, ask them to substantiate them."

Grade 7:

- Unit 3, Measuring Circles, Lesson 7: Exploring the Area of a Circle, Activity 7.4, Cool-Down - Areas of Two Circles, Responding to Student Thinking ,provides guidance for teachers responding to student answers and provides feedback focusing on the lessons most important activities. "1. More Chances. Students will have more opportunities to understand the mathematical ideas in this Cool-Down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this Cool-Down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding."
- Unit 5, Rational Number Arithmetic, Lesson 2: Changing Temperatures, Activity 2.1, Which One Doesn't Belong: Arrows, encourages students to compare four number line diagrams with arrows while analyzing them carefully before they have to interpret them in terms of rational number arithmetic. Launch, "Arrange students in groups of 2–4. Display the image of the four figures for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason each figure doesn't belong." Student Response, "1. The only one where both arrows point right. 2. The only one where the arrows point in opposite directions and are different lengths. 3. The only one where the arrows point in opposite directions and are the same length. 4. The only one where both arrows point left." Teachers are provided guidance on how to respond to the data through Activity Synthesis. "After students have conferred in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which diagram does not belong, attend to students' explanations and ensure the reasons given make sense. Ask the students what they think the arrows might represent. After collecting responses, say we are going to represent positive and negative numbers and their sums using arrows on a number line."
- Unit 9, Putting It All Together, Lesson 10: Measuring Long Distances Over Uneven Terrain, Activity 10.3, Comparing Methods, Anticipated Misconceptions, provides evidence around gathering, analyzing, using, and communicating language and content data from the activity. Students use a previously taught method to measure the length of a path chosen by the teacher. "Some students may need to be reminded how to use the measuring tools accurately, such as starting at the 0 mark and keeping the measuring tool going in a straight line." Student Response, "1. Answers vary. 2. Answers vary. Sample response: Report the average between the two measurements. 3. Answers vary. Sample responses: If group A's

measurement is 50 m and group B's measurement is 51 m, then group B's measurement is 2% larger than group A's since 51

- ÷ 50 = 1.02. 4. Answers vary. For the two methods given in previous task: a. Use a measuring tape over and over again. Advantages: Can be very accurate. Disadvantage: It takes two people and is quite cumbersome. If not done carefully, each time the tape moves, an error is introduced. So this is not very practical for long distances and if there are a lot of corners to go around. b. Measure stride length, and then count the number of strides. Advantages: Very easy to do and very quick. Disadvantage: Not all strides are equal. The longer the distance, the more chances for errors there are." Activity Synthesis provides guidance around student responses, "Invite the different groups to share their solutions. Ask them to: Compare how close their answers are. Compute the approximate relative error (difference/total length). Discuss the advantages and disadvantages of their methods and sources of discrepancies in their measurements, and how a small error can propagate. The takeaway should include: We can use proportional reasoning to find longer distances. If we know it takes 10 steps to walk 8 meters, then it will take 20 steps to walk 16 meters. Small errors can magnify over longer distances. Methods were either not very precise (prone to introduce error), or they were precise but cumbersome to implement."

Grade 8:

- Unit 5, Functions and Volume, Lesson 5: More Graphs of Functions, Activity 5.1, Which One Doesn't Belong: Graphs, encourages students to notice and describe the features of graphs using their own language. Launch, "Arrange students in groups of 2–4. Display the image of the four graphs for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. Follow with a whole-class discussion." Student Response, "Answers vary. Sample response: A doesn't belong because it is the only graph that touches the horizontal axis. B doesn't belong because it is the only one that is not a function. C doesn't belong because it is the only one made of straight line segments or because it is the only graph with no interval where it is decreasing. D doesn't belong because it is the only one made of discrete points or because it is the only graph with two distinct intervals where it is decreasing." Teachers provide guidance on how to respond to the data through Activity Synthesis. "After students have conferred in groups, invite each group to share one reason why a particular graph might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which graph does not belong, attend to students' explanations and ensure the reasons given are correct. Try to highlight the two facts that there are points with the same first coordinate and different second coordinates in graph B (which means it is not a function), the straight segments of C vs. curves of the others, and the discrete nature of D during this discussion, using whatever language students bring to it. Avoid introducing the traditional x and y names for the axes into the discussion unless students use them first. More formal vocabulary will be developed

in later activities, lessons, and grades, and much of the motivation of this added vocabulary is to improve upon the somewhat clunky language we are led to use without it."

- Unit 7, Exponents and Scientific Notation, Lesson 1: Exponent Review, 1.4, Cool-Down - Exponent Check, Responding to Student Thinking, provides guidance for teachers responding to student answers and provides feedback focusing on the lessons most important activities. "3. Press pause. If students struggle with this Cool-Down, working with exponents, make time to revisit the work of Grade 6: Unit 6 Working with Exponents. See the Course Guide for ideas to help students re-engage with earlier work."
- Unit 9, Putting It All Together, Lesson 6: Using and Interpreting a Mathematical Model, Activity 6.2, Interpreting a Mathematical Model, Anticipated Misconceptions, provides evidence around gathering, analyzing, using, and communicating language and content data from the activity. Students analyze a model and make a prediction. "Students may want to say "For every one unit increase in x, y decreases by 1.07 units." Ask them to use the specific units and quantities in the model, latitude in degrees north and temperature in degrees Fahrenheit." Student Response, "1. For every degree latitude moving north, the temperature decreases by 1.07°F. 2. Vertical: The temperature at 0 degrees north (that is, on the equator) is 119°F. Horizontal: A latitude where high temp is 0°F would have to be over 100 degrees north, which doesn't exist. 3. Latitudes are restricted to 0–90 degrees by the situation. Factors other than latitude influence temperature, and those factors seem to be more important close to 0 degrees and close to 90 degrees. The model only used cities in North America. It should not be used to make predictions about temperatures on other continents without checking similar data there first." Activity Synthesis provides guidance around student responses, "Invite students to share their responses. Discuss the limitations and uses of the model. Consider asking the following questions: "What are some limitations of the model?" "Do limitations mean that the model is not good?" (No, it just means that we have to be aware of when we can use it and when we can't use it. Our model is pretty good for latitudes between 25 and 65 degrees north, and for locations in North America.) "What questions do you have about predicting temperature?" "How could you extend your investigation of predicting temperature or the weather?"